As described earlier, the reduction in heating at separation indicates that the flow remains laminar up to that point; however, it is well known that separated flows are particularly prone to transition in the shear layer.[6] In this case, transition is most likely at the higher flap angles and could reduce the separation length while substantially increasing the heat flux at reattachment.[7] Transition therefore could help explain the observed discrepancies between experimental results and numerical predictions. Similarly, the presence of Görtler-like vortical structures in the reattachment zone also must be considered a possibility. Evidence for such structures, both in the initially laminar interactions reported here and in higher-Reynolds-number flows where the interaction is known to be turbulent, has been obtained in experiments involving liquid crystal thermography.[1,8] The observed spacing of these structures was such that it is quite possible that at least one crossed each gauge in this region, causing an increase in the local heat flux.

In an attempt to resolve these differences, numerical modeling of this type of flowfield at a higher Mach number and a lower unit Reynolds number[9] was carried out. Under these conditions, the flowfield is more likely to remain laminar and free from three-dimensional effects. The preliminary results obtained (not reported here) were compared with experimental and computational results reported in Ref. 9. In this case, our numerical results slightly overpredict the experimental heat fluxes but agree well with the other computational results, supporting our view that unaccounted-for three-dimensional and/or transition effects were present in the Southampton experiments. Further developments of the code are being planned to allow the investigation of flowfields involving these phenomena as well as other, more complex, three-dimensional features in hypersonic flows.

Conclusions

Numerical simulations of a laminar, two-dimensional flat-plate/compression-ramp hypersonic flow have been carried out. It appears that the general flow features, which include a large region of separated flow, are modeled correctly but the separation and reattachment lengths are slightly overpredicted. Although good agreement with experimental heat flux data is apparent on the flat-plate surface, the heat flux is underpredicted on the ramp and, in particular, in the reattachment area. Reasons for the discrepancies are discussed, and it is concluded that three-dimensional effects, possibly accompanied by transition in the free shear layer, are probably responsible for the observed differences between numerical and experimental results.

Acknowledgments

This work was supported by the United Kingdom Engineering and Physical Science Research Council. R. A. East of the Department of Aeronautics and Astronautics, University of Southampton, and P. Batten of the University of Manchester Institute of Science and Technology are thanked for their advice.

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Greatest Critical Difference Statistics in *k*-out-of-*n* Reliability Structures

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Introduction

AS the new millennium approaches, there is increasing expectation that routine but limited space travel will be a reality. Travel beyond the solar system is likely to remain for now a very distant idea due to the well-known physical laws or the way such laws are understood at this time. Regardless of the nature of space travel, risk is a fundamental consideration. There is often a negative relationship between the cost and the risks of the mission. If sufficient risk is accepted, a mission may cost a fraction of the amount needed for a much less risky one. As travel distances get larger, risk analysis becomes even more relevant. A new probability tool, distribution of the greatest critical difference of component lives or simply the range distribution, is suggested, possibly for the first time in reliability literature. This study considers a reliability concern of spacecraft, given that each of their engines has a life that is a random variable with known or estimated parameters. If each engine has a random life at some specified operating conditions, such as speed and temperature, then the probability of failure and the failure rate will increase over time in the absence of preventive maintenance. Decision makers can install a fixed number n of such engines, some or all of which are needed to keep the spacecraft flying. Assume that it is possible to keep the spacecraft flying if one or more of the engines have failed. The proposed methodology can be applied to spacecraft structure scenarios in various decision stages, especially, for example, the prephase A stage used in NASA's decision process. In a 4-out-of-4 system (where all engines must be operable for the spacecraft to fly), the system life distribution is the same as the distribution of the minimum life component. On the other hand, the 1-out-of-4 system (where only one engine is required to power the spacecraft) has a life distribution that is the same as the distribution of the maximum life component. Sarper[1] has shown analytical and simulation results for the distribution of the extremes in the four-engine case. This study shows analytical solutions for range analysis using the 1-out-of-4 case. Then simulation is used and is validated against the analytical solution. This validation is used in suggesting that simulation-only results for other cases should be valid too.

Received June 27, 1997; revision received Dec. 4, 1997; accepted for publication Dec. 4, 1997.

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Sample Reliability Structure

The system consists of four independent engines or components. A mean life of 3000 time units and two distributions are assumed. An exponential distribution is commonly used in reliability work for illustration purposes because this distribution has a constant failure rate irrespective of duration of use. The second distribution is uniform to represent the case when the failure rate increases over time as it would in a long and nonstop space voyage. The mean life of 3000 is maintained by assuming lower and upper limits of 2500 and 3500, respectively. Let t_i equal the random life of the ith unit in the system ($i = 1, 2, 3, \ldots, n$), $t_{[j]}$ equal the life of the unit that fails in the jth sequence ($j = 1, 2, 3, \ldots, n$), and $T_{k,n}$ equal the life of the k-out-of-n system, where $k = 1, 2, 3, \ldots, n$, or

$$T_{k,n} = t_{[n-(k-1)]}$$

For example, let the random lives be 2601.21, 3416.40, 3200.15, and 2829.92 time units. Then $T_{1,4} = t_{[4-(1-1)]} = t_{[4]} = t_2 = 3416.40$. Similarly, $T_{2,4} = 3200.15$, $T_{3,4} = 2829.92$, and $T_{4,4} = 2601.21$.

Greatest Critical Difference Statistic

This is the new tool proposed in this Note. The greatest critical difference or range is a rare statistic not discussed in most textbooks. Gumbel's[2] textbook is one major source of information. Also, others[3,4] have discussed this topic in detail. Range is the duration between the first failure and the last critical failure and is a random variable of great significance. This statistic provides an important mechanism with which long space flights can be better planned. For instance, if a 1-out-of-4-type spacecraft is under consideration, the mission may be considered in jeopardy if full power is not available. In the preceding example, the range is $3416.40 - 2601.21 = 815.19$ because the last critical failure for a 1-out-of-4 system is failure number 4. As longer range is desirable, the spacecraft has 815.19 time units to arrange for repair work and/or return after the mission is aborted. Then $R_{k,n}$ is the range of the k-out-of-n structure:

$$R_{k,n} = T_{k,n} - T_{n,n} = T_{k,n} - t_{[1]} \tag{1}$$

where $t_{[1]}$ is the minimum life. Range gets shorter as the last critical failure becomes engine number $n-1$, $n-2$, and so on. Another definition of range is a duration of flight in distress and is equal to zero for n-out-of-n systems. Range should be described as a probability distribution function (PDF) or, at least, parameters of its distribution should be known to assist in planning work to determine the number of engines to install. Standby engines are not considered in this analysis, but range information would be useful in determination of how many standby engines, if any, should be considered once all engine reliability and cost information are available.

Analytical Results

The preceding example may be valid for just one of the many identical spacecraft that will be traveling in the future. To make policy decisions, the analyst needs probability models that describe the behavior of all such spacecraft. Analytical results are always preferable to simulation because analytical formulas are applicable to any input data. However, simulation has a distinct advantage of always providing the decision maker with a PDF of the output of interest. But this is often not possible when analytical work is complicated. Analytical expressions for PDFs and other parameters of k-out-of-4 systems are available in Refs. 1 and 5. In the case of uniform distribution, the minimum possible value for the range of n engines is zero if each engine life t can take any value from a to b (where $0 \le a < b$): $f(t) = 1/(b-a)$, from a to b, inclusive, and 0 elsewhere. The maximum value is $b - a$ or 1000 in the sample problem regardless of the number of engines. The cumulative density function (CDF) is needed to obtain the PDF. Let s be the random variable range:

$$F(s) = P\{S \le s\} = P\{r(b-a) \le S\}$$
$$= P\{r \le S/(b-a)\} = F_r\{S/(b-a)\} \tag{2}$$

To get the PDF of the range, function F_r needs to be differentiated with respect to S to get $f(s)$

$$f(s) = n(n-1)[1/(b-a)]^{n-1} S^{n-2}[1 - S/(b-a)]$$
$$\text{for} \quad a \le S \le (b-a), \quad a \le b \tag{3}$$

where n is the number of engines.

The general PDF expression (3) can be used to obtain expressions for CDF, mean, median, and variance of the range random variable for four engines or a 1-out-of-4 structure. The CDF in the same interval as in Eq. (3) is

$$-\frac{S^3(4a + 3S - 4b)}{(a-b)^4} \tag{4}$$

Expression (4) is equal to 0 at $s = 0$ and is equal to 1 at $s = b - a$, indicating that it is a proper CDF. The median can be found by setting CDF equal to 0.5 and finding a root that falls in the same interval. The expressions for mean and standard deviations are

$$\text{mean} = -\frac{3(a-b)}{5} \tag{5}$$

$$\text{variance} = \frac{(a-b)^2}{25} \tag{6}$$

The case when n is 4 is considered. No analytical derivations are attempted for the range distribution for 2- or 3-out-of-4 structures. Their analytical derivations would be more complicated than the steps just used. Combinatorial considerations must be included in the derivation process. If each identical engine has a life distributed exponentially with rate λ (inverse of the mean life), $f(t) = \lambda\exp(-\lambda t)$, where t is nonnegative. This PDF ranges from zero to infinity, and the minimum/maximum-range values should be also zero and infinity. Again, let s be the sample range of n exponentials in which V is the largest observation. It is possible to use the following general formula[6] to derive a PDF for the range:

$$f(s) = \int_{-\infty}^{+\infty} n(n-1)[F(V) - F(V-s)]^{n-2} f(V-s) f(V)\,\mathrm{d}V \tag{7}$$

The function F in Eq. (7) denotes the CDF of each exponential distribution. The general equation (7) reduces to

$$f(s) = (n-1)\lambda[\exp(-\lambda s) - 1]^{n-2}\exp[-\lambda s(n-1)], \qquad s \ge 0 \tag{8}$$

and for the $n = 4$ case,

$$f(s) = 3\lambda[\exp(-\lambda s) - 1]^2 \exp(-3\lambda s), \qquad s \ge 0 \tag{9}$$

If the PDF (9) is integrated from zero to some intermediate value s, the following CDF expression is obtained:

$$F(s) = 1 - 3\exp(-\lambda s) + 3\exp(-2\lambda s) - \exp(-3\lambda s)$$
$$s \ge 0 \tag{10}$$

Expression (10) is equal to 1 if evaluated at infinity and 0 at 0, indicating that it is a valid CDF for the exponential case. Unlike the uniform case, the general PDF expression (9) cannot be integrated unless a value is chosen for n. Similarly, it is harder to apply standard probability formulas to obtain general expressions for mean and variance without also specifying a value for λ. Integrating $[s\ \ f(s)]$ and $[s^2\ \ f(s)]$ expressions for general λ is difficult, but one can perform numerical integration once $\lambda = 1/3000$ is specified. Then mean and standard deviation are found as 5500 and 3500 time units, respectively.

Simulation Experiments

Simulation was performed to confirm the analytical parameter expressions and also to illustrate that simulation is an excellent alternative to analytical work. Table 1 shows the results of 26,000 (default) iterations. Simulation results are easily obtained for the other two reliability structures not investigated. A commercial Monte Carlo simulation package[7] was used. Each simulated output was analyzed using a chi-square, goodness-of-fit test to identify the best PDF to represent the output. Table 2 shows the best-fitting distributions. For the 1-out-of-4 case using uniformly distributed random variables, true values of mean, maximum, minimum, standard deviation, and median are 600, 1000, 0, 200, and 614.28, respectively. Table 1 shows corresponding simulation results 599.05, 999.38, 19.93, 199.84, and 610.27, respectively. Table 2 shows that simulated data can be described by a Weibull distribution with scale and shape parameters of 667.10 and 3.45. The analytical CDF is as given by Eq. (4). The simulated CDF is

$$1 - \exp\left[-\left(\frac{s}{667.10}\right)^{3.45}\right] \qquad \text{for} \qquad s \geq 0 \qquad (11)$$

Expression (11) is equal to zero at $s = 0$ but is equal to 0.982, not 1.0, at $s = 1000$. This discrepancy may explain the inaccuracies observed in Table 1. All minimum and maximum values must be 0 and 1000 for all of the structures shown in Table 1. If the number of simulation trials is increased well beyond the software maximum of 26,000, most of the inaccuracies can be avoided. True or analytical parameters for mean, maximum, minimum, standard deviation, and median are 5500, infinity, 0, 3500, and 4735.25, respectively, when each engine of the 1-out-of-4 structure is exponentially distributed. Corresponding simulation results (Table 1) are 5499.91, 39,462.89, 72.46, 3521.36, and 4719.86, respectively. The minimum simulation value of 72.46 is close enough to zero, but the maximum value, 39,462.89, is far from infinity. It can be shown that this discrepancy does not really invalidate the value of the simulation process and its accuracy. The maximum simulation value can approach infinity if the number of simulation trials is well in excess of the software limit of 26,000 or one can perform a huge number of runs, each with a maximum trial size of 26,000. This argument, when complemented with both means being almost the same, proves that simulation is a valid alternative to complex analytical work. Table 2 shows that the simulated data are best described by an extended gamma distribution function with a CDF of

$$1 - \exp\left(-\frac{s - 156.75}{2239.81}\right)\sum_{j=0}^{1}\left(\frac{s - 156.75}{2239.81}\right)\bigg/ j! \qquad s \geq 156.75 \qquad (12)$$

The shape parameter of 2.41 is rounded down to 2.0 to be able to write out expression (12). A gamma function of an integer is needed to obtain a closed-form gamma CDF.

Table 1 Simulation results for range distribution

Component life	Parameter	1-out-of-4 range	2-out-of-4 range	3-out-of-4 range
Exponential	Mean	5,499.91	2,505.71	998.71
	Maximum	39,462.89	18,122.47	11,133.95
	Minimum	72.46	11.79	1.64 E-2
	Standard deviation	3,521.36	1,809.99	1,004.02
	Median	4,719.86	2,082.31	698.44
Uniform	Mean	599.05	399.64	200.82
	Maximum	999.38	983.34	917.48
	Minimum	19.93	1.97	0.02
	Standard deviation	199.84	200.38	164.57
	Median	610.27	383.43	159.22

Table 2 Best-fitting distribution of the range of *k*-out-of-4 structures

Component life	1-out-of-4 range	2-out-of-4 range	3-out-of-4 range
Exponential	Gamma (E)[a]	Gamma (E)	Gamma (E)
Location parameter	156.75	22.24	0.12
Scale parameter	2239.81	1337.28	1060.61
Shape parameter	2.41	1.87	0.96
Uniform	Weibull	Weibull	Weibull
Location parameter	0	0	0
Scale parameter	667.10	453.41	212.53
Shape parameter	3.45	2.17	1.16

[a]E, extended version.

Applications

What is the probability that the range will be at least 600 time units using uniformly distributed engine lives? Equation (4) yields a CDF value of 0.4752. Its complement, 0.5248, is the answer. The approximate answer is 0.4997 using Eq. (11). What is the interpretation of this information? A 1-out-of-4 spacecraft has a 52.48% probability of having at least 600 time units of flight capability left once the first engine has failed. Simulation results underestimate this probability as 49.97%. As there are no analytical formulas developed for the other two structures, simulation-based CDF descriptions (Table 2) must be used for 2- and 3-out-of-4 structures: The same probability drops to 15.94% and 3.57% for these structures, respectively. The reduced probabilities are expected as the last critical engine becomes engines 3 and 2, respectively. Equation (10) yields a probability of 99.41% for at least 600 time units of range using exponential distribution. Although both uniform and exponential distributions have the same mean of 3000, the exponential distribution has much longer range parameters for the 1-out-of-4 case. Simulation-based CDF (12) again underestimates this probability as 98.28%. This is still very accurate despite the shape parameters of 2.41 having to be rounded down to 2.0. Even without the CDF information, analytical or simulation based, one can use the simulation-based parameter data (Table 1) to make various probability statements on the random variable of interest (bounds, intervals, etc.) by applying probability theorems such as Chebyshev's inequality.

Conclusion

To maintain the continuing pace of pioneering planetary missions, spacecraft must be highly reliable. This study has provided some simple probability and simulation-based results and concepts that can be used in this effort. A new statistic, range, was proposed. It was shown that the range statistic can play an important role in determination of spacecraft design for very long trips. The analytical expressions presented are simple, but none has been found readily available in relevant literature. The closeness of simulated and analytical parameters is encouraging for the value of simulation, which is simply an experiment. Simulation becomes an indispensable tool if the underlying random variables are correlated and/or described by harder-to-manipulate PDFs.

Acknowledgments

This work is supported by the Colorado Space Grant Consortium. We are grateful for director Wolfang Sauer's assistance.

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Efficient Jitter Analysis for Spacecraft

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Introduction

TYPICALLY in space missions, the science instruments require a specific degree of pointing accuracy as well as dynamical quietness. This dynamical quietness, which is needed to allow the instruments to make measurements (remote sensing applications) or to perform other functions, is usually characterized in terms of jitter and stability specifications.[1,2] To ensure that the spacecraft meets the requirements of its instruments, several jitter analyses are performed throughout the design phase of the spacecraft and beyond as the models of the spacecraft, its components, and disturbances mature. Each such analysis involves the simulation of the spacecraft and instrument dynamical response to all known disturbance scenarios, followed by the computation of jitter values for each instrument based on the specified jitter time windows.[1,2] The direct approach for computing jitter values by sweeping maxima and minima throughout the time history may be costly in the computational sense as the size and number of the time histories involved could be quite large. Keeping in mind that typical spacecraft simulation time histories may easily involve hundreds of thousands or millions of points, it is imperative that a jitter analysis algorithm that is more efficient than the direct approach be developed. This Note presents a vectorized algorithm for efficient computation of spacecraft jitter values. The algorithm identifies the extreme points in the time history, which are the points that may dominate the jitter values depending on the location of the jitter window along the time history. The span of influence of each extremum is then computed by the algorithm and used in an efficient and vectorized fashion to obtain the jitter values. The algorithm deals with the multiple jitter windows sequentially, first computing jitter values for the smallest time window and then looping over the remaining time windows until all jitter values are computed. A numerical example is carried out to demonstrate the efficiency and feasibility of the proposed jitter analysis technique.

Problem Formulation

Let $\boldsymbol{y}(t)$ represent the time history of the spacecraft response at a specified location on the spacecraft due to some disturbances. Assume $\boldsymbol{y}$ has n elements corresponding to equally stepped time values with a time increment of T. Furthermore, assume that jitter values are desired for several jitter time windows represented by the elements of $\boldsymbol{t}_j$, where $\boldsymbol{t}_j(i) < \boldsymbol{t}_j(i+1)$, $i = 1, 2, \ldots, m$, with m denoting the number of jitter time windows in $\boldsymbol{t}_j$.

The jitter value for the time window $\boldsymbol{t}_j(i)$ is defined as maximum peak-to-peak excursions of the spacecraft output response, within a time window of size $\boldsymbol{t}_j(i)$ seconds, over all possible positions of such a window in the response time history. Reference 3 provides a complete definition of jitter.

Jitter value is a measure of the level of spacecraft motion in on-orbit conditions, as well as, to some degree, the frequency content of such motion, depending on the size of the jitter window. The direct approach for computing jitter would involve performing $n - k + 1$ maximum and minimum operations on vectors of size k, where k, which denotes the number of points in a given jitter window, is given by

$$k = fix\{\boldsymbol{t}_j(i)/T\} + 1 \tag{1}$$

The direct approach may be computationally acceptable if the jitter window is very small (k is near 1) or very large (k is near n). However, it is quite inefficient for most realistic problems where the size of the jitter window is not at either extreme and the size of the time history can be in the hundreds of thousands or millions. An efficient algorithm for the computation of jitter/stability has been developed. This algorithm is based on vector operations to achieve a drastic speedup of the computational time over the direct approach. The algorithm starts with some preprocessing of the time history data and then loops through the requested jitter windows, starting from the smallest. The algorithm is described in the following sections. Note that most of the calculations in the algorithm can be carried out by vector arithmetic operations that can be executed more rapidly in an array processing language such as MATLAB[4] or on a vector or parallel processing computer than by executing a loop performing scalar calculations.

Step 1: Identification of Extrema

In the first step, the extrema in the entire time history are identified. These include maxima, minima, and level response points. The motivation behind this is that, for any given jitter window position along the time history, the jitter value is dominated by either extrema (maxima, minima, or level response points) in the window, if they are present, and/or by the response at the endpoints of the window. To identify the extrema in the time history, compute the first difference vector of the time history array y:

$$\boldsymbol{y}_1 = \boldsymbol{y}(2:n) - \boldsymbol{y}(1:n-1) \tag{2}$$

where $\boldsymbol{y}(2:n)$, for example, defines a vector formed from the elements 2 through n of y. Now, compute the sign difference vector for the first difference vector $\boldsymbol{y}_1$:

$$\boldsymbol{s}_1 = \mathrm{sgn}[\boldsymbol{y}_1(2:n-1)] - \mathrm{sgn}[\boldsymbol{y}_1(1:n-2)] \tag{3}$$

The locations associated with the internal maxima may be determined as

$$\boldsymbol{l}_{\max} = \{i+1 \mid \boldsymbol{s}_1(i) = -1 \text{ or } -2\} \tag{4}$$

Here, ones are added to the indices to identify points in the original time history and not the sign difference history. Similarly, the locations associated with the minima may be determined as

$$\boldsymbol{l}_{\min} = \{i+1 \mid \boldsymbol{s}_1(i) = 1 \text{ or } 2\} \tag{5}$$

Let $\boldsymbol{y}_{\max}$ and $\boldsymbol{y}_{\min}$ represent the identified maxima and minima in the time history, corresponding to locations in vectors $\boldsymbol{l}_{\max}$ and $\boldsymbol{l}_{\min}$, respectively. It is assumed that the elements of vectors $\boldsymbol{l}_{\max}$ and $\boldsymbol{l}_{\min}$ are in ascending order. This comprises the necessary preprocessing of the time history data that must be performed at the beginning of the procedure. The remaining steps in the procedure are applied sequentially for each jitter time window, starting from the smallest.

Step 2: Reduction of Extrema

Not all extrema contribute to the computation of jitter value for a given time window. For example, a maximum that is surrounded by larger maxima on the left and the right would not contribute to the jitter value if the distance, in time, between the surrounding maxima is not greater than the given time window. A similar argument also applies to the minima, with the exception that the surrounding minima should be smaller.

Received July 15, 1997; revision received Nov. 28, 1997; accepted for publication Jan. 5, 1998.

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